Packet #2

This unit covers graphing trigonometric functions along with simplifying and proving identities.

NOTes

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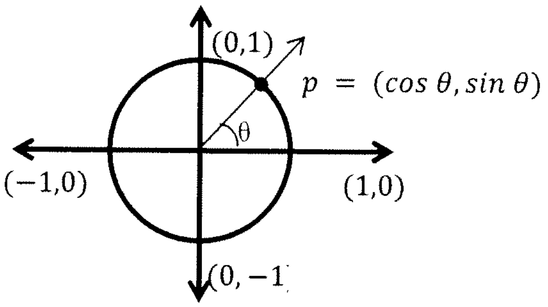
TUTORING SERVICES

How to Graph Trigonometric Functions

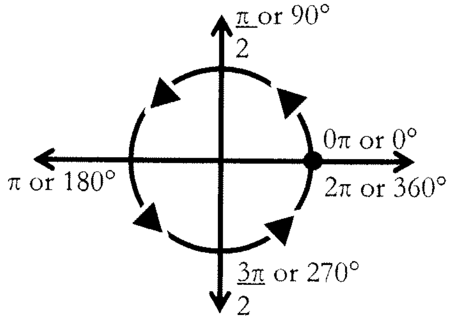
This handout includes instructions for graphing processes of basic, amplitude shifts, horizontal shifts, and vertical shifts of trigonometric functions.

The Unit Circle and the Values of Sine and Cosine Functions

The unit circle is a circle with a radius that equals 1. The angle θ is formed from the φ (phi) ray extending from the origin through a point p on the unit circle and the x-axis; see diagram below. The value of $sin\ \theta$ equals the y-coordinate of the point p and the value of $cos\ \theta$ equals the x-coordinate of the point p as shown in the diagram below.



This unit circle below shows the measurements of angles in radians and degrees. Beginning at 0π, follow the circle counter-clockwise. As angle θ increases to $\frac{\pi}{2}$ radians or 90°, the value of cosine (the x-coordinate) decreases because the point is approaching the y-axis. Meanwhile, the value of sine (the y-coordinate) increases. When one counter-clockwise revolution has been completed, the point has moved 360° or 2π.



Graphing Sine and Cosine Functions $y = \sin x$ and $y = \cos x$

There are two ways to prepare for graphing the basic sine and cosine functions in the form $y = \sin x$ and $y = \cos x$: evaluating the function and using the unit circle.

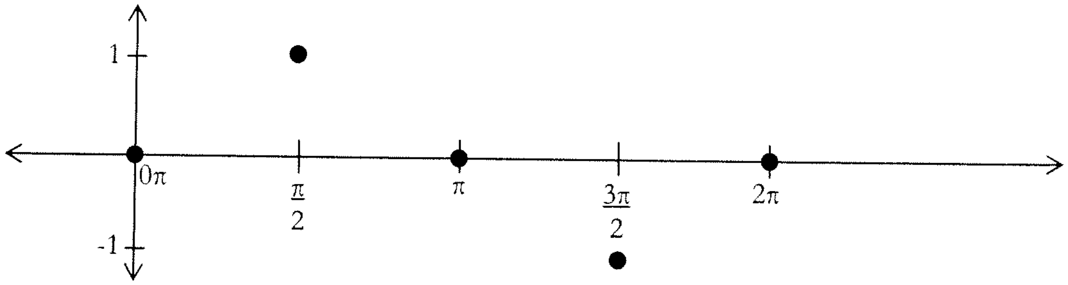
To evaluate the basic sine function, set up a table of values using the intervals $0\pi, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π for x and calculating the corresponding y value.

$f(x)$ or $y = \sin x$

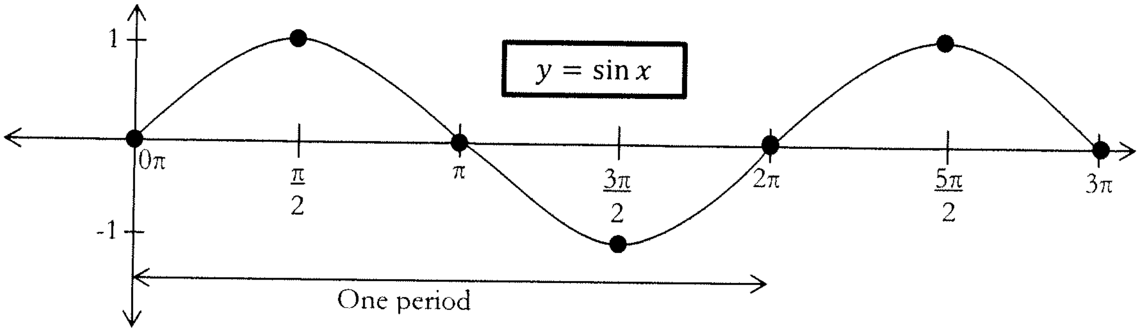
$f(x)$ or y	x
0	0π
1	$\frac{\pi}{2}$
0	π
-1	$\frac{3\pi}{2}$
0	2π

To use the unit circle, the x-coordinates remain the same as within the list above. To find the y-coordinate of the point to graph, first locate the point p on the unit circle that corresponds to the angel θ given by the x-coordinate. Then, use the y-coordinate of the point p as the y value of the point to graph.

To draw the graph of one period of sine or $y = \sin x$, label the x-axis with the values $0\pi, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π. Then plot points for the value of $f(x)$ or y from either the table or the unit circle.



Other points may be added for the intermediate values between those listed above to obtain a more complete graph, and a best fit line can be drawn by connecting the points. The figure on the next page is the completed graph showing one and a half periods of the sine function.



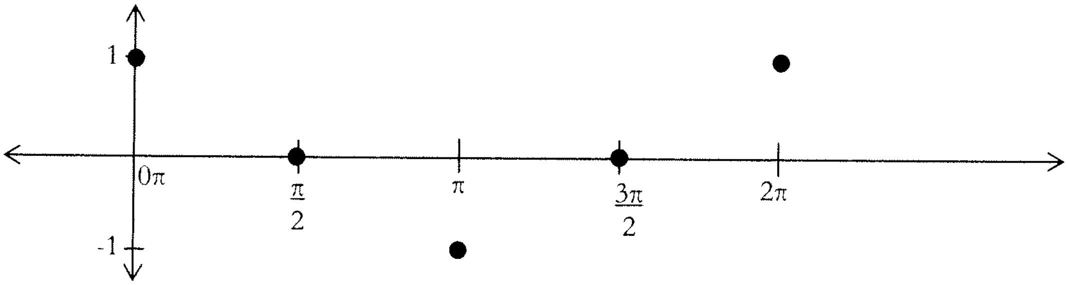
The graph of the cosine function $y = \cos x$ is drawn in a similar manner as the sine function. Using a table of values:

$f(x)$ or $y = \cos x$

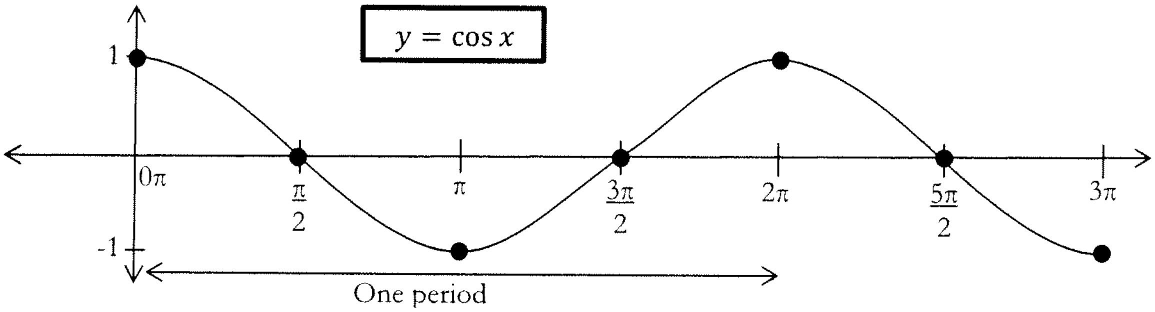
$f(x)$ *or* y	x
1	0π
0	$\frac{\pi}{2}$
-1	π
0	$\frac{3\pi}{2}$
1	2π

To use the unit circle, the x-coordinate remains the same as the list on the previous page. To find the y-coordinate of the point to graph, first locate the point p on the unit circle that corresponds to the angel θ given by the x-coordinate. Then, use the x-coordinate of the point p as the y value of the point to graph.

To draw the graph of one period of cosine or $y = \cos x$, label the x-axis with the values 0π, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π. Then plot points for the value of $f(x)$ or y from either the table or the unit circle.

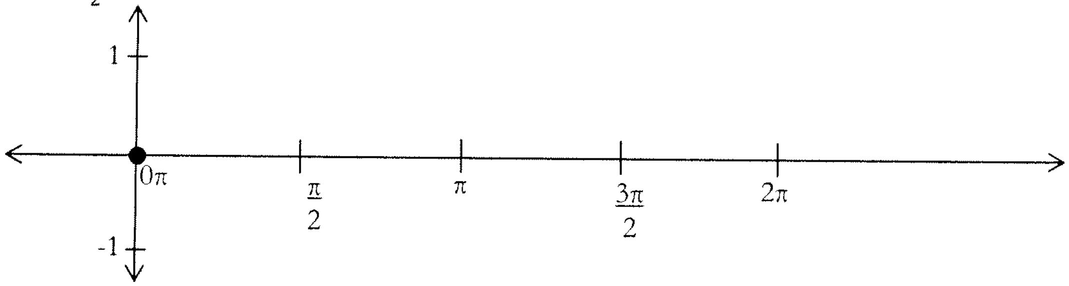


Add other points as required for the intermediate values between those above to obtain a more complete graph, and draw a best fit line connecting the points. The graph below shows one and a half periods.

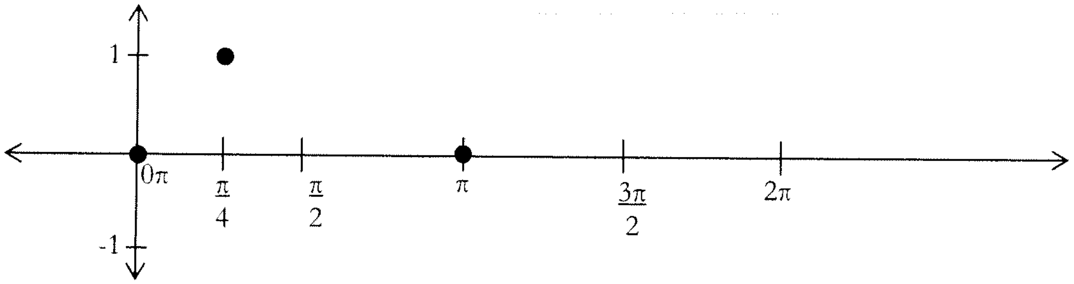


Graphing the Tangent Function $y = tan\, x$

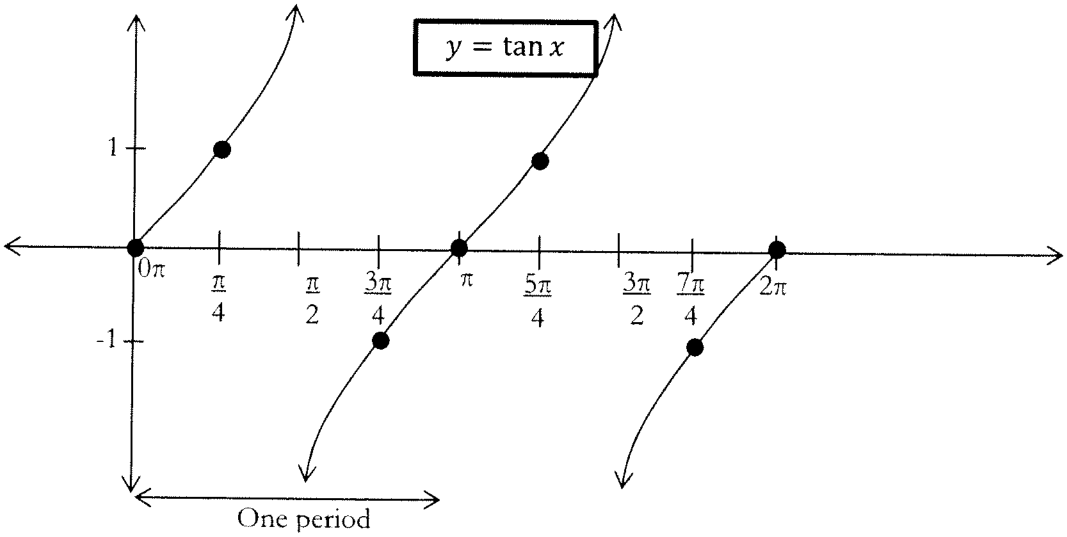
The tangent value at angle θ is equal to the sine value divided by the cosine value $\left(\frac{Sine\ Value}{Cosine\ Value}\right)$ of the same angle θ. The value of tangent at 0π for the unit circle is $\frac{0}{1}$, which is equivalent to 0. The value of tangent at $\frac{\pi}{2}$ is $\frac{1}{0}$. This yields a divide by 0 error or undefined (try this in your calculator). Therefore, the tangent function is undefined at $\frac{\pi}{2}$. This is illustrated by drawing an asymptote (vertical dashed line) at $\frac{\pi}{2}$. See the figure below.



The value of tangent at π is $\frac{0}{1}$, which results in 0. To determine how the tangent behaves between 0π and the asymptote, find the sine and cosine values of $\frac{\pi}{4}$, which is half way between 0π and $\frac{\pi}{2}$. Looking at the handout *Common Trigonometric Angle Measurements*, the tangent of $\frac{\pi}{4}$ is $\frac{\sqrt{2}}{2}$ (sine) divided by $\frac{\sqrt{2}}{2}$ (cosine). Flipping the cosine value and multiplying gives: $\frac{\sqrt{2}}{2} \times \frac{2}{\sqrt{2}}$ which simplifies to 1. The value of tangent at $\frac{\pi}{4}$ is therefore 1. These points have been added to the graph below.



Next, calculate the value of tangent for $\frac{3\pi}{4}$. Consulting with *Common Trigonometric Angle Measurements,* the tangent of $\frac{3\pi}{4}$ is $\frac{\sqrt{2}}{2}$ (sine) divided by $-\frac{\sqrt{2}}{2}$ (cosine). This simplifies to a tangent value of -1. Now, draw the tangent function graph so that the line approaches the asymptote without touching or crossing it. The image on the next page shows the completed graph of one and a half periods of the tangent function.



The period of the basic tangent function is π, and the graph will repeat from π to 2π.

The Form $y = A\sin(Bx + C) + D$

The form $y = A\sin(Bx + C) + D$ is the general form of the sine function. From this general form of the sine function, the amplitude, horizontal, phase, and vertical shifts from the basic trigonometric forms can be determined.

A : modifies the amplitude in the y direction above and below the center line

B : influences the period and phase shift of the graph

C : influences the phase shift of the graph

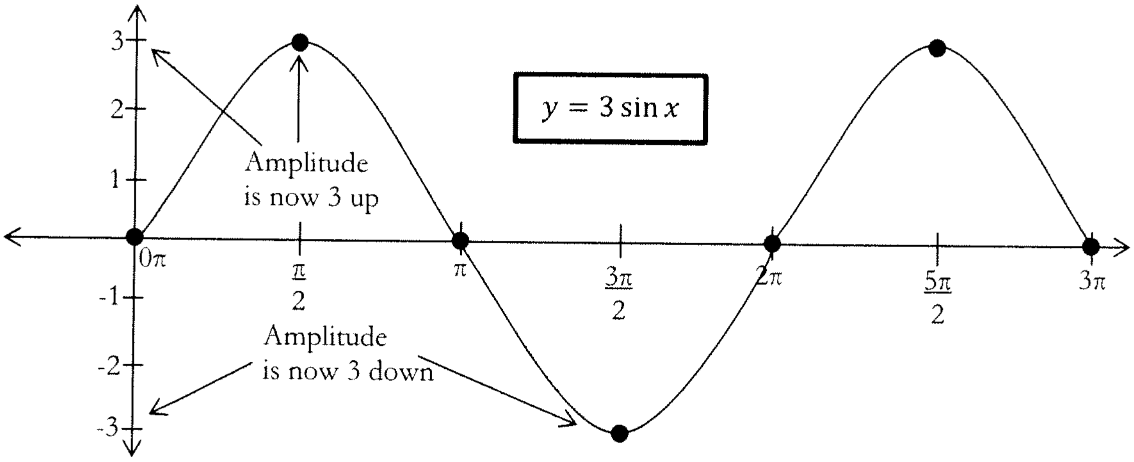
D : shifts the center line of the graph on the y-axis

Amplitude Shifts of Trigonometric Functions

The basic graphs illustrate the trigonometric functions when the A value is 1. This $A = 1$ is used as an amplitude value of 1. If the A value is not 1, then the absolute value of A value is the new amplitude of the function. Any number $|A|$ greater than 1 will vertically stretch the graph (increase the amplitude) while a number $|A|$ smaller than 1 will compress the graph closer to the x axis.

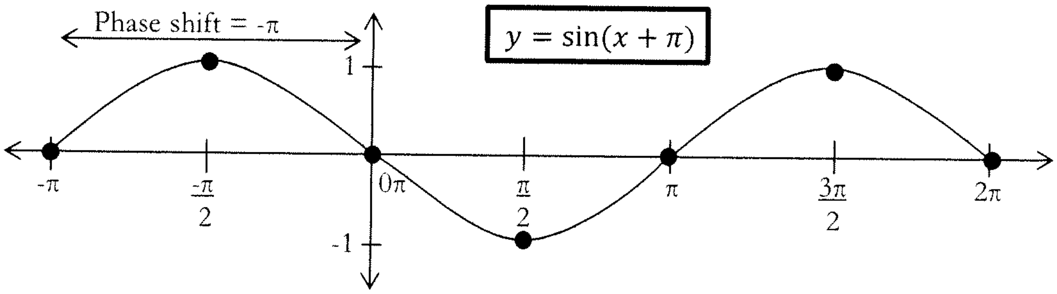
Example: Graph $y = 3\sin x$.

Solution: The graph of $y = 3\sin x$ is the same as the graph of $y = \sin x$ except the minimum and maximum of the graph has been increased to -3 and 3 respectively from -1 and 1.



Horizontal Shifts of Trigonometric Functions

A horizontal shift is when the entire graph shifts left or right along the x-axis. This is shown symbolically as $y = \sin(Bx - C)$. Note the minus sign in the formula. To find the **phase shift** (or the amount the graph shifted) divide C by $B\left(\frac{C}{B}\right)$. For instance, the phase shift of $y = \cos(2x - \pi)$ can be found by dividing π (C) by 2 (B), and the answer is $\frac{\pi}{2}$. Another example is the phase shift of $y = \sin(-2x - \pi)$ which is $-\pi$ (C) divided by -2 (B), and the result is $\frac{\pi}{2}$. Be careful when dealing with the signs. A positive sign takes the place of the double negative signs in the form $y = \sin(x + \pi)$. The C is negative because this example is also written as $y = \sin(x - (-\pi))$, which produces the negative π phase shift (graphed below). It is important to remember a positive phase shift means the graph is shifted right or in the positive direction. A negative phase shift means the graph shifts to the left or in the negative direction.



Period Compression or Expansion of Trigonometric Functions

The value of B also influences the period, or length of one cycle, of trigonometric functions. The period of the basic sine and cosine functions is 2π while the period of the basic tangent function is π. The period equation for sine and cosine is: Period $= \frac{2\pi}{|B|}$. For tangent, the period equation is: Period $= \frac{\pi}{|B|}$. Period compression occurs if the absolute value of B is greater than 1; this means the function oscillates more frequently. Period expansion occurs if the absolute value of B is less than 1; this means the function oscillates more slowly.

The starting point of the graph is determined by the phase shift. To determine the key points for the new period, divide the period into 4 equal parts and add this part to successive x values beginning with the starting point.

Summer Supplement: Graphing Trig. Functions Unit Name:__________________________

Lesson #1: The Sine Graph

Web Resources

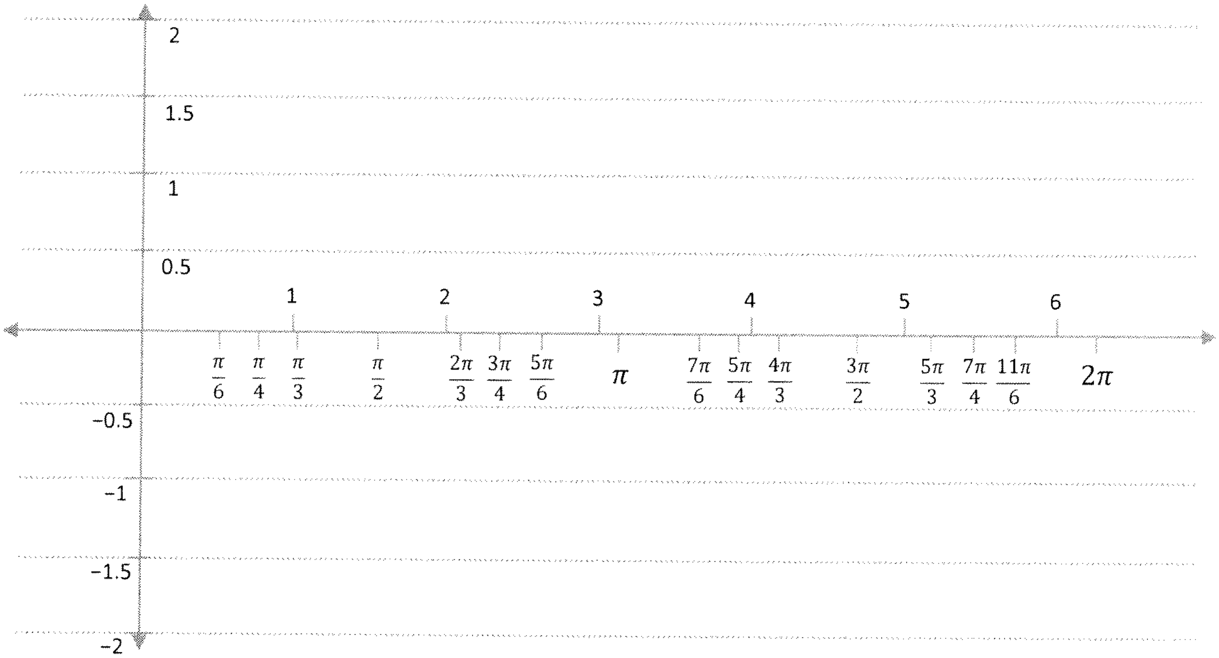
Khan Academy: https://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/graphs-of-sine-cosine-tangent/v/we-graph-domain-and-range-of-sine-function

Youtube: https://www.youtube.com/watch?v=qQOKUIrcuRs

1. $y = \sin x$ a) Fill-in the table with the fractional values for the equation, $y = \sin x$, then use a calculator to help approximate the decimal value for each of the fractions. After you have filled in the table of values, plot each of the points on the graph paper below and connect the dots with a smooth flowing curve.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$y = \sin x$ *(fractional)*									
$y = \sin x$ *(decimal)*									

x	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$y = \sin x$ *(fractional)*									
$y = \sin x$ *(decimal)*									



What you have just graphed is one period of the function $y = \sin x$.

b) What is the highest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

c) What is the lowest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

d) At what locations (x-coordinate) is the value of the function (y-coordinate) equal to zero? _______________

The ***amplitude*** for the curve $y = \sin x$ is 1, since it goes up one and down one from its equilibrium value of elevation zero. (By the way, amplitude is always a non-negative value.)

2. $y = 2\sin x$ a) Think about what the 2 would do to each of the y-values from the filled-in table for $y = \sin x$. Using a different color from the previous problem, plot the points appropriate to $y = 2\sin x$ on the same graph, then connect the dots with a smooth flowing curve and answer the following questions.

b) What is the highest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

c) What is the lowest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

d) At what locations (x-coordinate) is the value of the function (y-coordinate) equal to zero? _______________

e) What is the amplitude for $y = 2\sin x$? _____

3. $y = \frac{1}{2}\sin x$ a) Think about what the ½ would do to each of the y-values from the filled-in table for $y = \sin x$. Using a different color from the previous problems, plot the points appropriate to $y = \frac{1}{2}\sin x$ on the same graph, then connect the dots with a smooth flowing curve and answer the following questions.

b) What is the highest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

c) What is the lowest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

d) At what locations (x-coordinate) is the value of the function (y-coordinate) equal to zero? _______________

e) What is the amplitude for $y = \frac{1}{2}\sin x$? _____

4. $y = -\sin x$ a) Think about what the -1 would do to each of the y-values from the filled-in table for $y = \sin x$. Using a different color from the previous problems, plot the points appropriate to $y = -\sin x$ on the same graph, then connect the dots with a smooth flowing curve and answer the following questions.

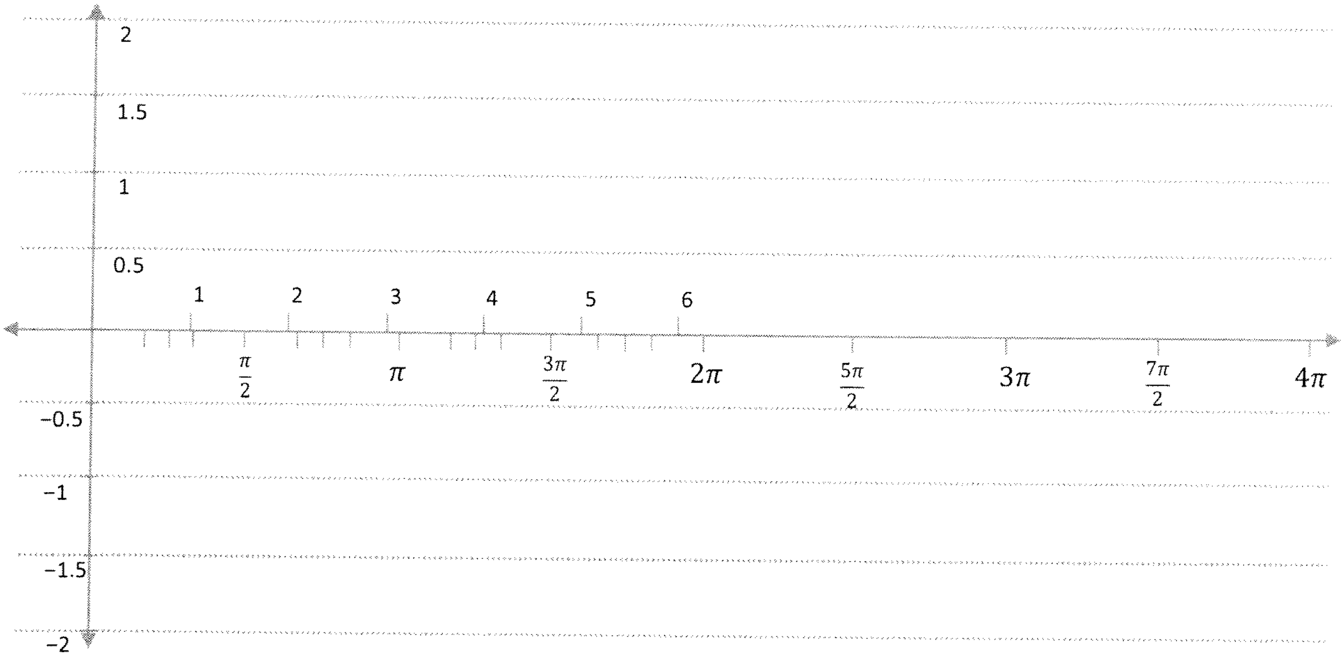
b) What is the highest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

c) What is the lowest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

d) At what locations (x-coordinate) is the value of the function (y-coordinate) equal to zero? _______________

e) What is the amplitude for $y = -\sin x$? _____

5. Sketch $y = \sin x$ for $0 \leq x \leq 4\pi$. Use what you have learned above and plug in decimal values between 2π and 4π to make it accurate.



Summer Supplement: Graphing Trig. Functions Unit

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Lesson #2: The Cosine Graph

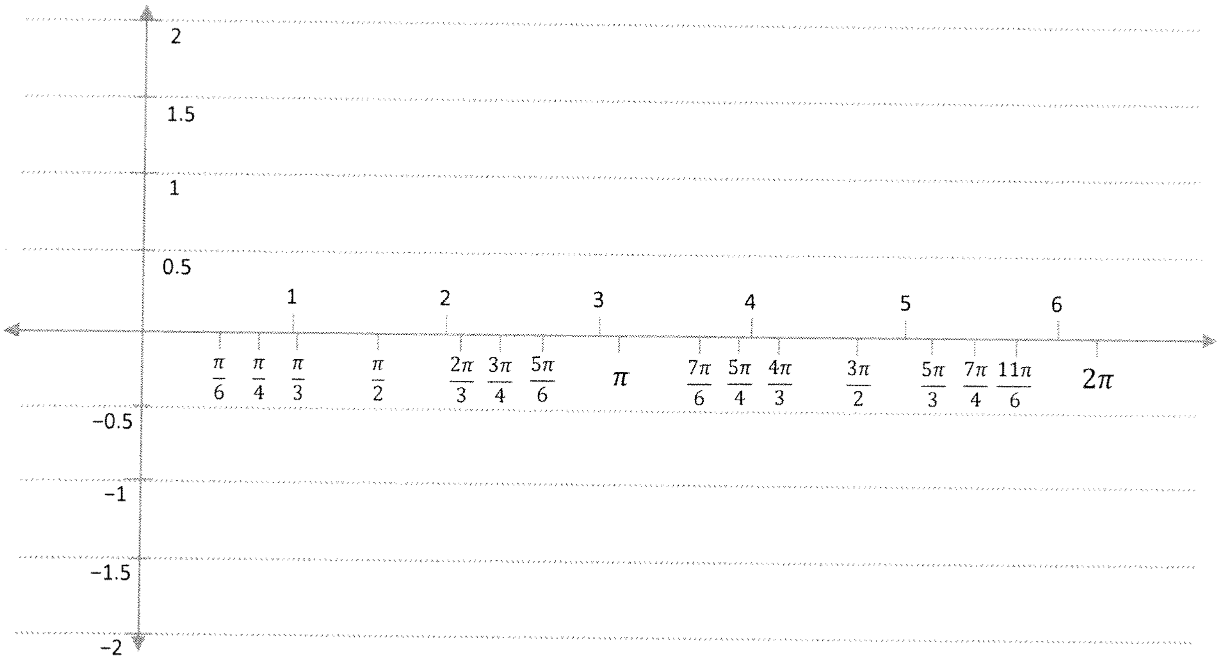
Web Resources

Youtube: https://www.youtube.com/watch?v=qQOKUIrcuRs

1. $y = \cos x$ a) Fill-in the table with the fractional values for the equation, $y = \cos x$, then use a calculator to help approximate the decimal value for each of the fractions. After you have filled in the table of values, plot each of the points on the graph paper below and connect the dots with a smooth flowing curve.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$y = \cos x$ *(fractional)*									
$y = \cos x$ *(decimal)*									

x	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$y = \cos x$ *(fractional)*									
$y = \cos x$ *(decimal)*									



What you have just graphed is one period of the function $y = \cos x$.

b) What is the highest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

c) What is the lowest value (y-coordinate) for this function? _____ At what location (x-coordinate) does this occur? _____

d) At what locations (x-coordinate) is the value of the function (y-coordinate) equal to zero? _______________

2. Graph the function $y = 2\cos x$. ***Use the graph above with a different color of curve.***

3. Graph the function $y = \frac{1}{2}\cos x$. ***Use the graph above with a different color of curve.***

4. Graph the function $y = -\cos x$. ***Use the graph above with a different color of curve.***

5. Without graphing, give the amplitude for each of the following cosine functions.

a) $y = -4\cos x$ b) $y = \pi\cos x$ c) $y = -\frac{3}{7}\cos x$ d) $y = a\cos x$

** Remember: One ***period*** of a trig function is one cycle, or the length needed to complete all of the values of the function before repeating the same pattern of values. ***What happens if we multiply the ANGLES by a constant?***

6. Sketch $y = \cos 2x$ (*at least two periods*) on the graph below.

Plug in the points from the table on the front side. Multiply the angle FIRST, then plug it into cosine.

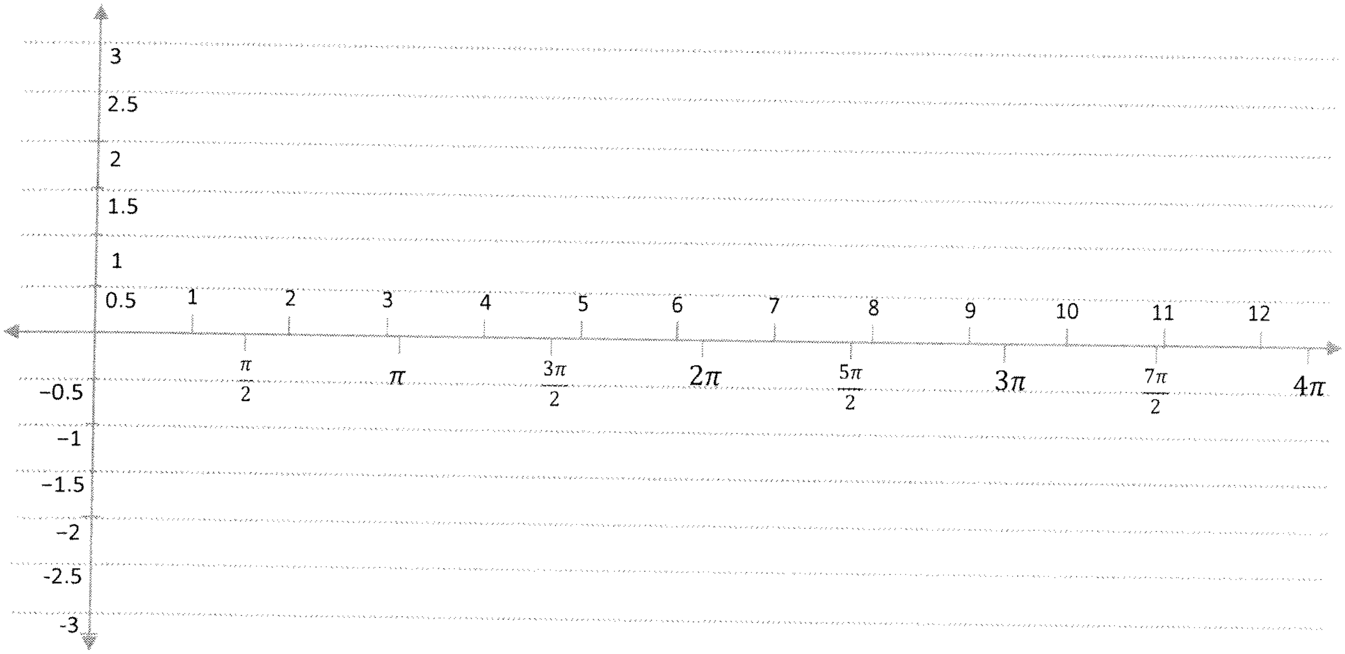
a) What is the amplitude? _____ b) What is the length of one period? _____

7. Sketch $y = -\frac{3}{2}\cos\frac{1}{2}x$ (*only one period*) on the graph below in a different color.

a) What is the amplitude? _____ b) What is the length of one period? _____

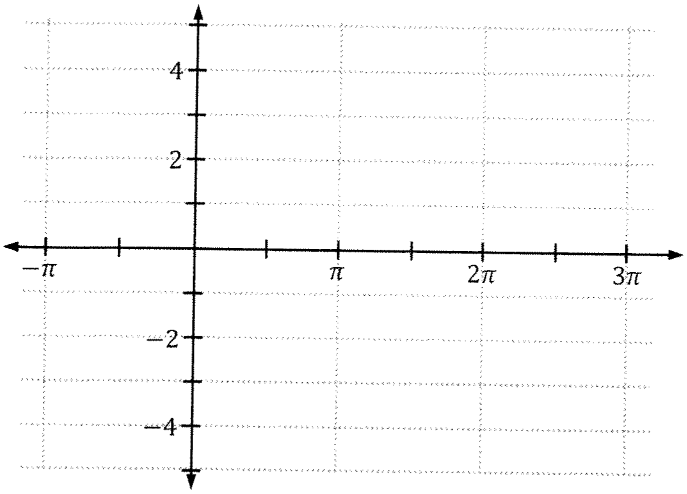
8. Sketch $y = 2\cos\frac{\pi}{2}x$ (*at least two periods*) on the graph below in a different color.

a) What is the amplitude? _____ b) What is the length of one period? _____

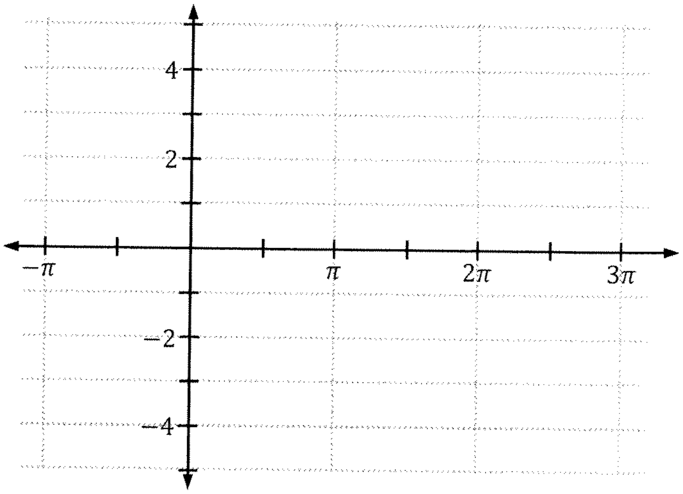


Each of the trig functions can be roughly sketched using 5 main points. Think about which 5 points those would be and then sketch each equation below using those five points. Pay attention to amplitude and period length. Include labels on the x- & y-axis.

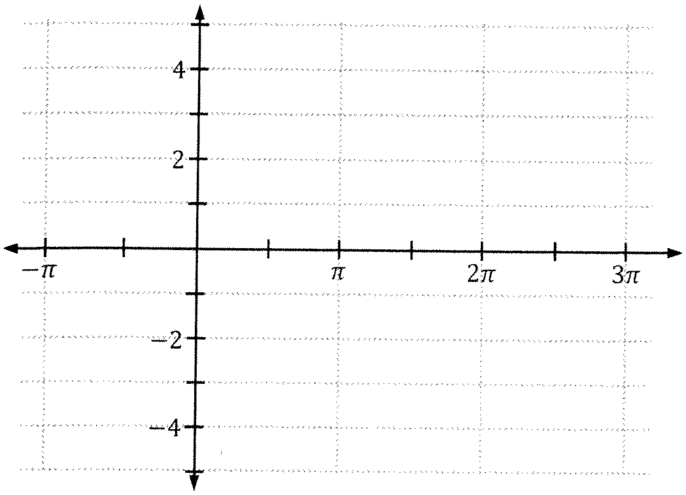
9. $y = -3\cos x$



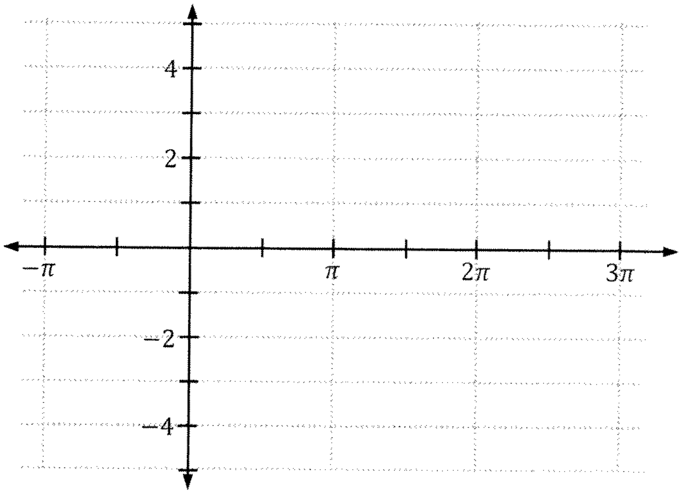
10. $y = 2\cos \pi x$



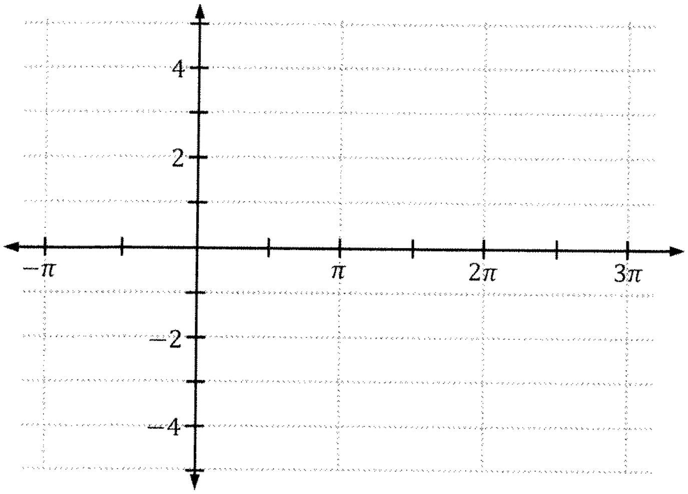
11. $y = \frac{1}{3}\cos 4x$



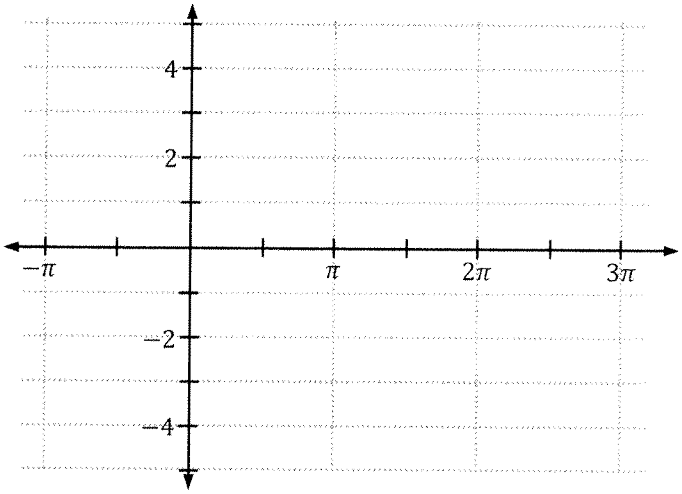
12. $y = 5\sin x$



13. $y = \frac{7}{2}\sin\frac{1}{2}x$



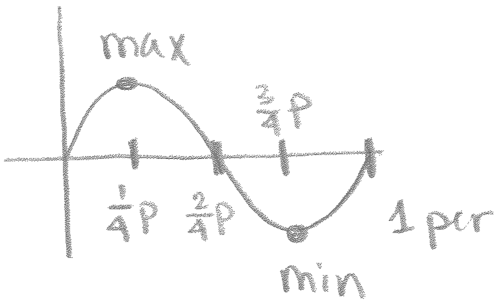
14. $y = -2\sin\frac{\pi}{4}x$



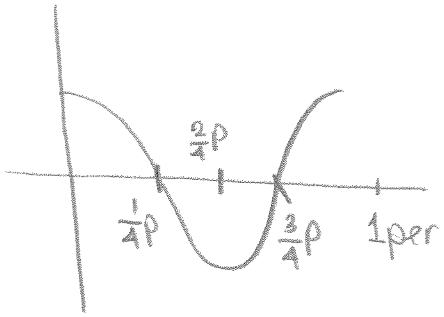
$y = A \sin Bx$ or $y = A \cos Bx$

Period: $\frac{2\pi}{B}$; Amplitude $= |A|$

$\boxed{\sin}$



$\boxed{\cos}$



Summer Supplement: Graphing Trig. Functions Unit

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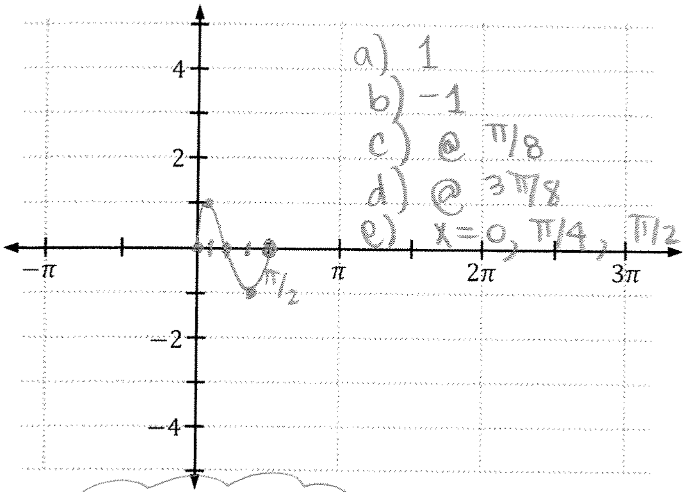
Lesson #3: Graphing Sine and Cosine: Notes

Sketch one period of the function. Label all critical points. Then give a) maximum height, b) minimum height, c) location of the maximum height, d) location of the minimum height, e) x-intercepts

1. $y = \sin 4x$



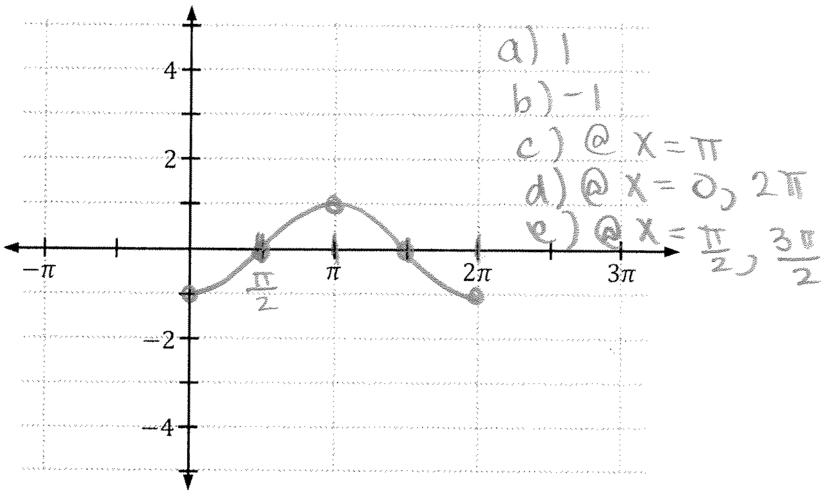
a) 1
b) −1
c) @ π/8
d) @ 3π/8
e) x = 0, π/4, π/2

$y = A \sin Bx$
$|A|$ = amplitude
Period = $\frac{2\pi}{B}$

; $y = \sin 4x$
; amp = 1
; Per = $\frac{2\pi}{4}$

Per = π/2

2. $y = -\cos x$

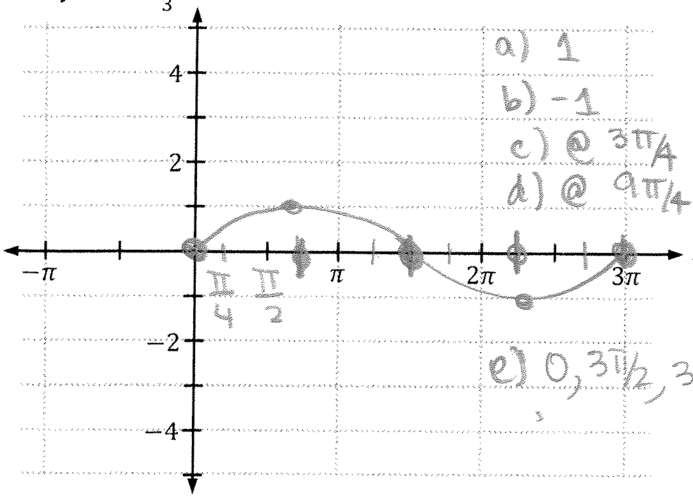


a) 1
b) −1
c) @ x = π
d) @ x = 0, 2π
e) @ x = $\frac{\pi}{2}$, $\frac{3\pi}{2}$

amp = $|-1| = 1$

per = $\frac{2\pi}{1} = 2\pi$

3. $y = \sin \frac{2}{3} x$



a) 1
b) −1
c) @ 3π/4
d) @ 9π/4
e) 0, 3π/2, 3π

$y = 1 \sin \frac{2}{3} x$

amp = $|1| = 1$

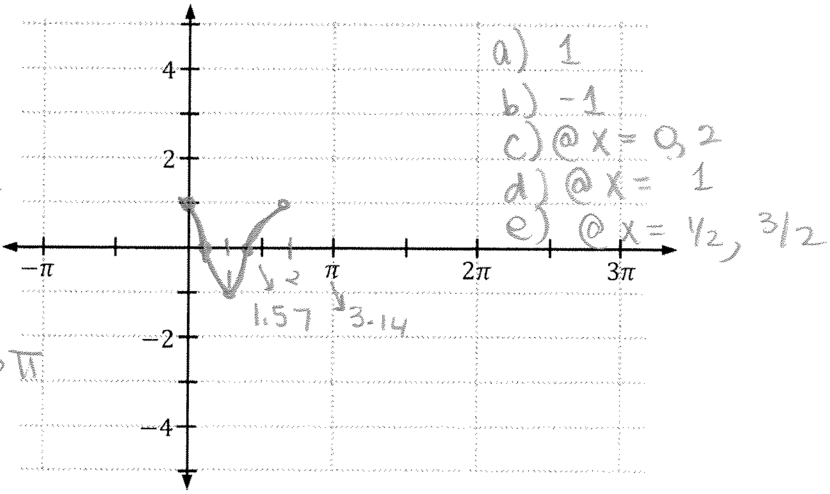
per = $\frac{2\pi}{2/3} \frac{3}{3} = \frac{6\pi}{2}$ or 3π

$\frac{1}{4}(3\pi) = 3\pi/4$

$2/4(3\pi) = 3\pi/2$

$3/4(3\pi) = 9\pi/4$

4. $y = \cos \pi x$



a) 1
b) −1
c) @ x = 0, 2
d) @ x = 1
e) @ x = 1/2, 3/2

$y = \cos \pi x$

amp = $|1| = 1$

per = $\frac{2\pi}{\pi} = 2$

$\frac{1}{4}(2) = 1/2 = .5$

$\frac{2}{4}(2) = 1 = 1$

$\frac{3}{4}(2) = 3/2 = 1.5$

Summer Supplement: Graphing Trig. Functions Unit
Lesson #3: Graphing Sine and Cosine

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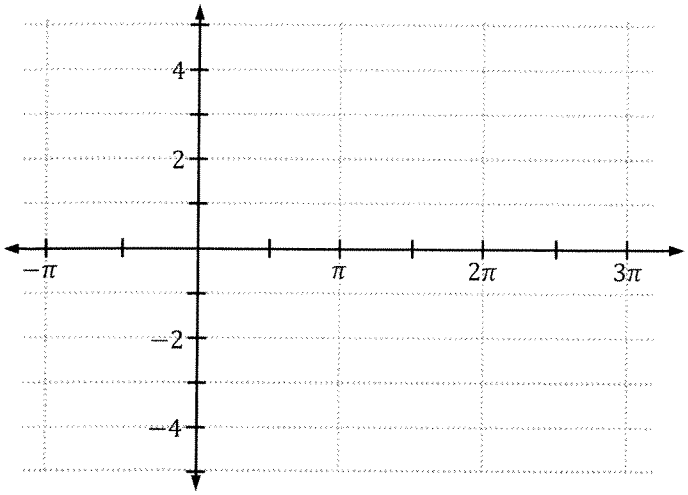
Web Resources

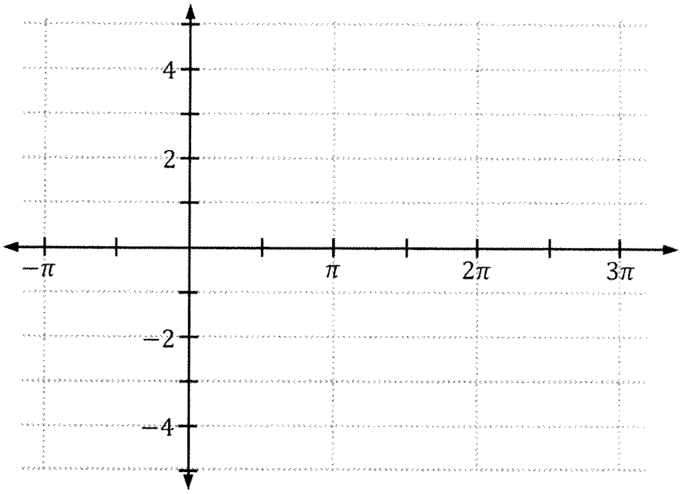
Patrick JMT: http://patrickjmt.com/graphing-sine-and-cosine-with-different-coefficients-amplitude-and-period-ex-1/

Sketch one period of the function. Label all critical points. Then give a) maximum height, b) minimum height, c) location of the maximum height, d) location of the minimum height, e) x-intercepts

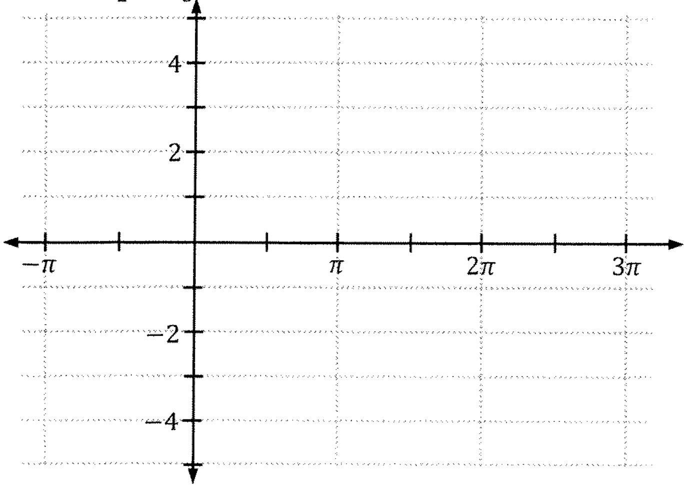
1. $y = \sin 2x$



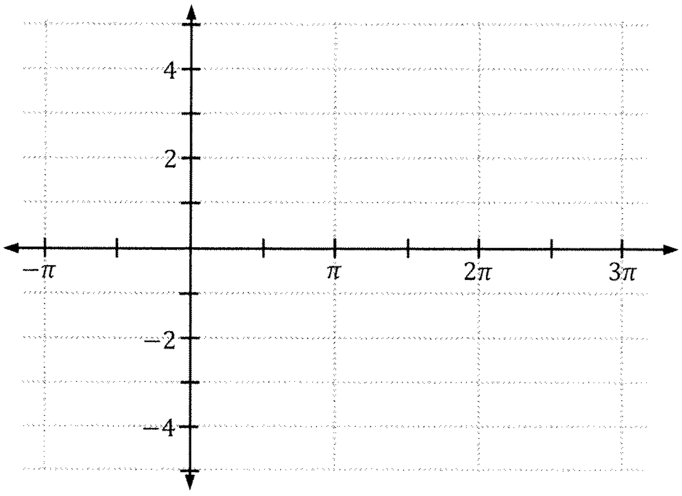
2. $y = -2\cos x$



3. $y = \frac{1}{2}\cos\frac{2}{3}x$



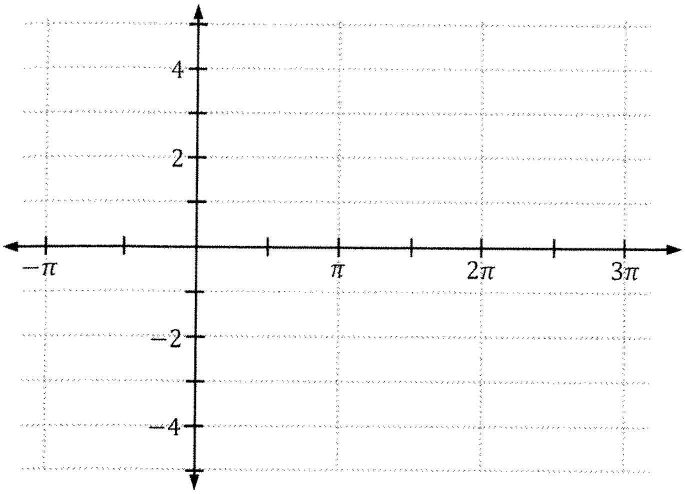
4. $y = \sin 2\pi x$

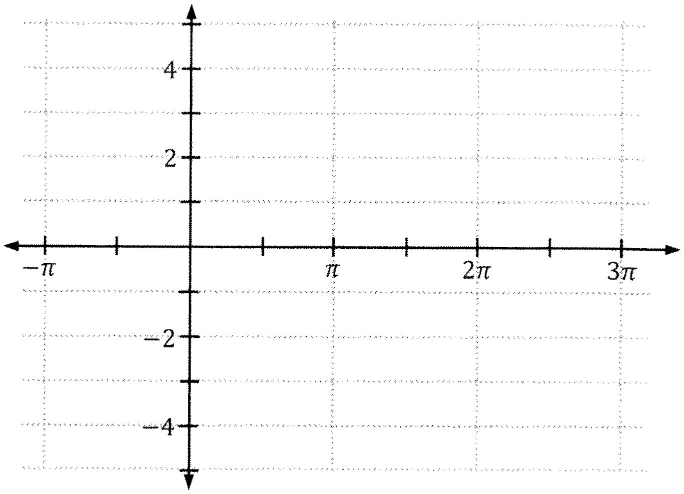


Sketch two periods of the function. Label all critical points.

5. $y = -\sin 4x$

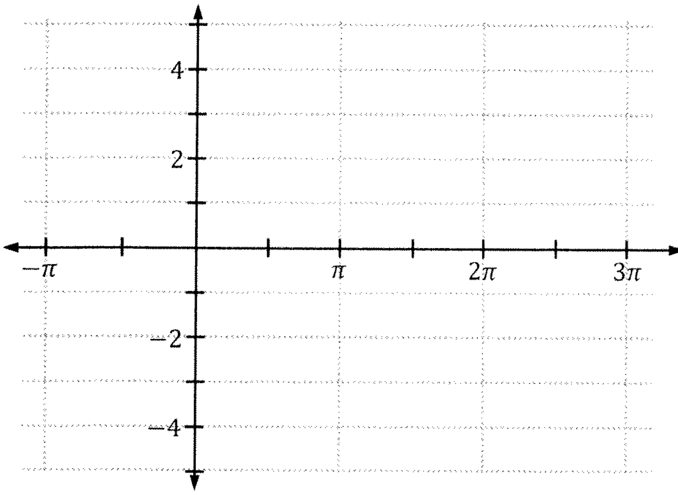
6. $y = 4\sin\frac{x}{3}$

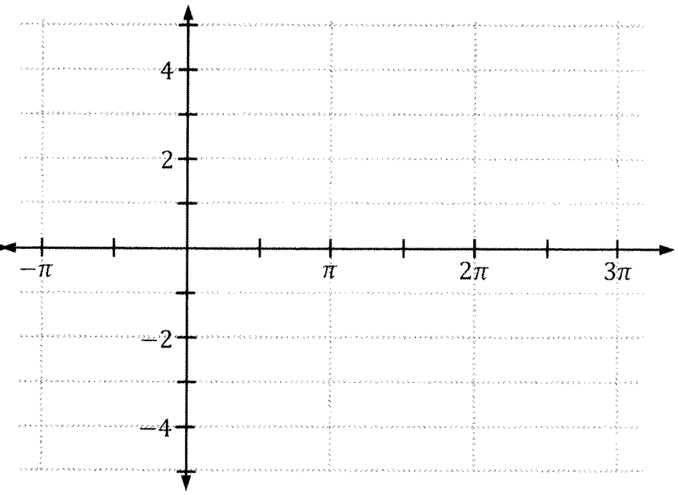




7. $y = -0.8\cos\frac{2}{3}x$

8. $y = \cos\frac{\pi}{4}x$



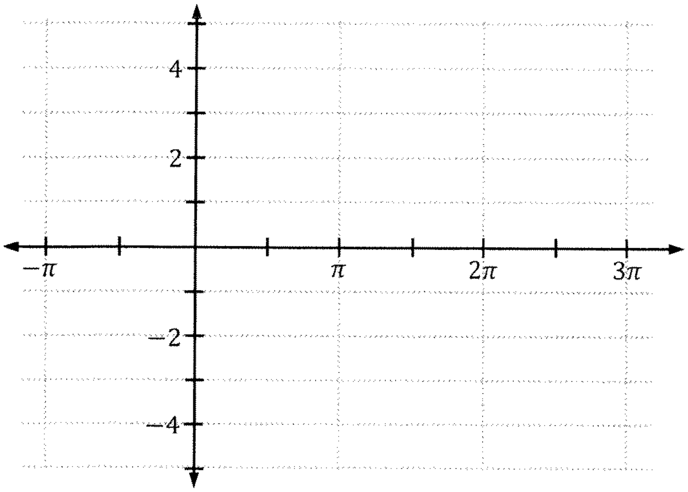


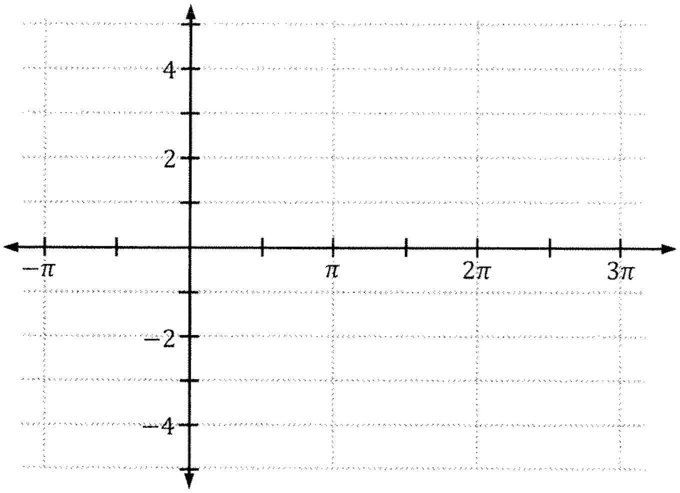
Sketch the function given on the given interval. Label all critical points.

hint: start sketching normally, then extend the graph to fill the interval

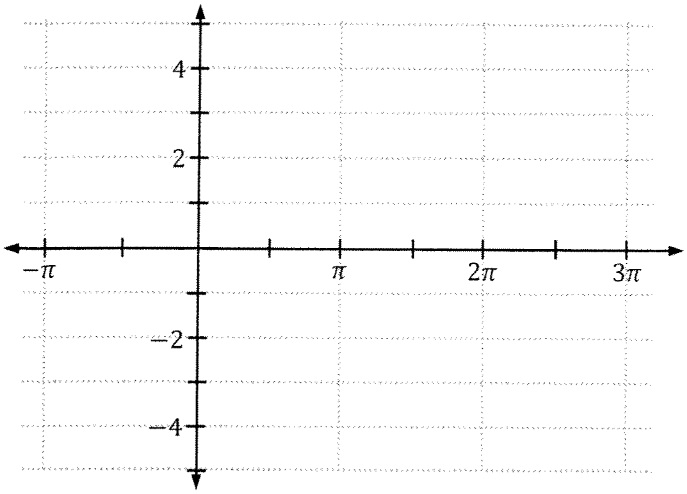
9. $y = \sin x, -\pi \leq x \leq 2\pi$

10. $y = 3\cos x, -\frac{3\pi}{2} \leq x \leq \pi$

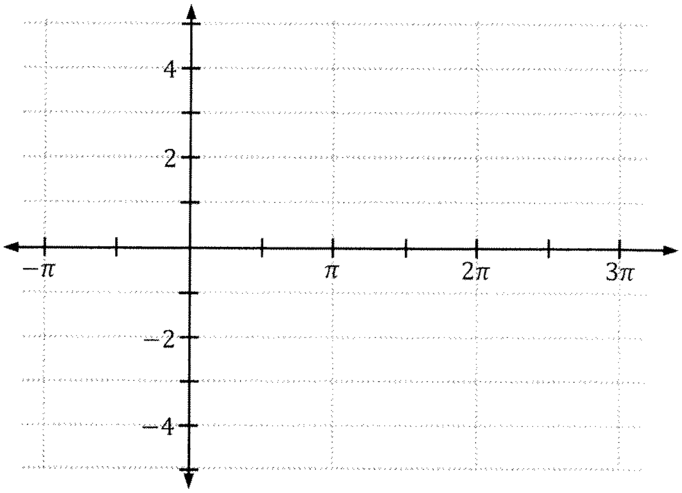




11. $y = -\frac{1}{2}\sin 3x, -\frac{2\pi}{3} \leq x \leq \frac{\pi}{3}$



12. $y = 2\cos \pi x, -3 \leq x \leq 5$



Summer Supplement: Graphing Trig. Functions Unit Name:______________________

Lesson #4: The Secant and Cosecant Graph

Web Resources

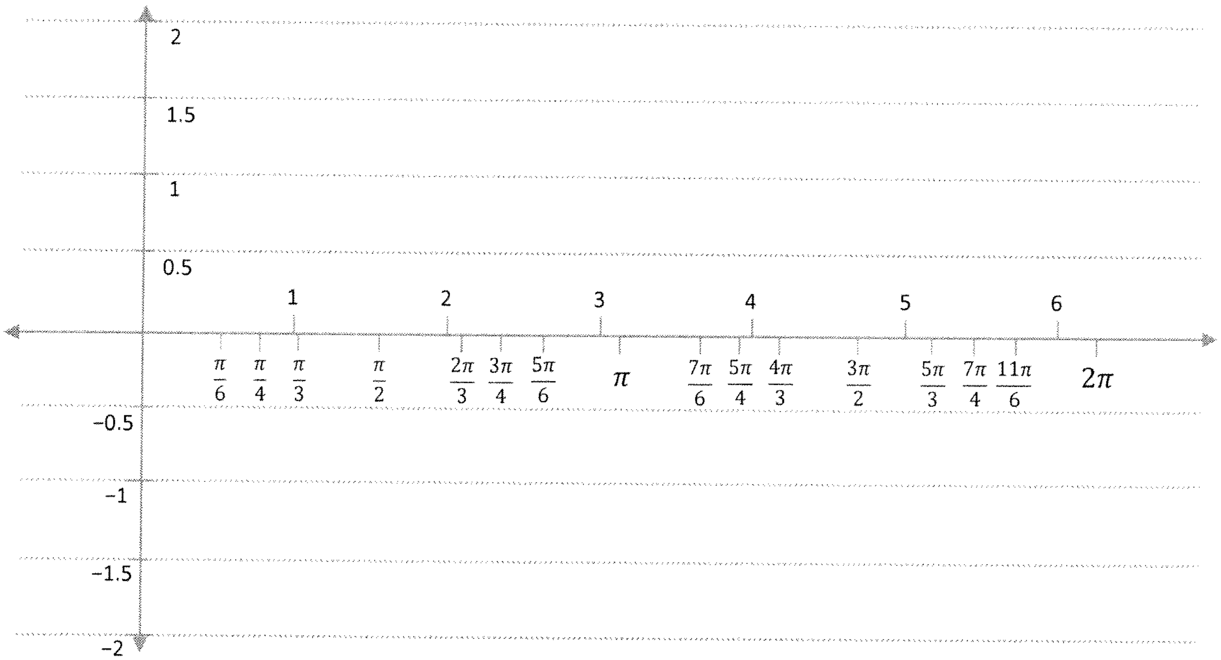
Patrick JMT: http://patrickjmt.com/graphing-the-trigonometric-functions/

Extra Resource: http://www.dummies.com/how-to/content/how-to-graph-a-secant-function.html

I. $y = \csc x$ a) Use a calculator to fill in the table with the decimal values for the equation, $y = \sin x$. Then fill in the values for $y = \csc x$ remembering that $\csc x$ is the reciprocal of $\sin x$. After you have filled in the table of values, *lightly* sketch $y = \sin x$ and then sketch $y = \csc x$.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$y = \sin x$ *(decimal)*									
$y = \csc x$ *(decimal)*									

x	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$y = \sin x$ *(decimal)*									
$y = \csc x$ *(decimal)*									



What you have just graphed is one period of the function $y = \csc x$. Answer the following questions:

b) Describe the behavior of $y = \csc x$ at the maximum value (y-coordinate) of $y = \sin x$. ______________________

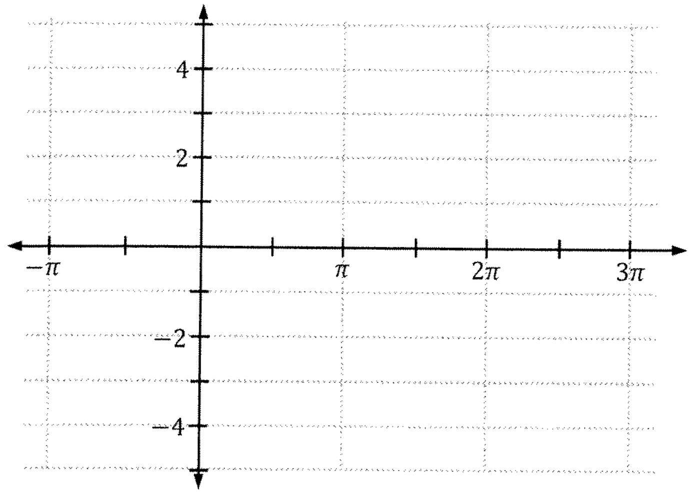
____

c) Describe the behavior of $y = \csc x$ when $y = \sin x$ crosses the x-axis. ______________________

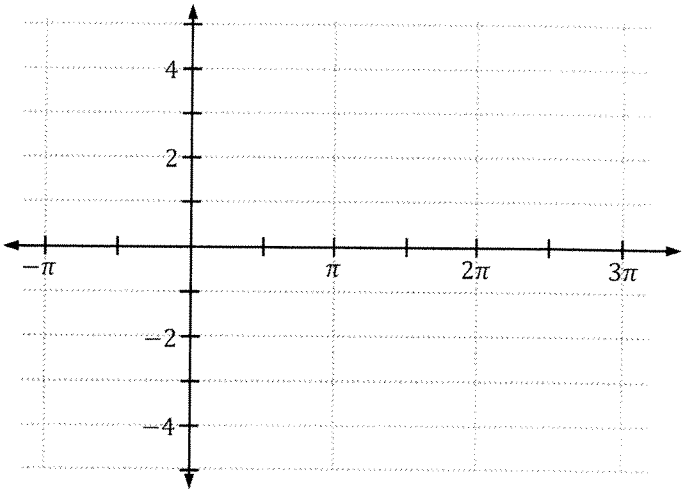
____

Practice: Sketch one period of each function using the critical points. Use a *lightly* drawn sine curve to help. Label all critical points. Remember how the amplitude and period adjustments work with $y = \sin x$.

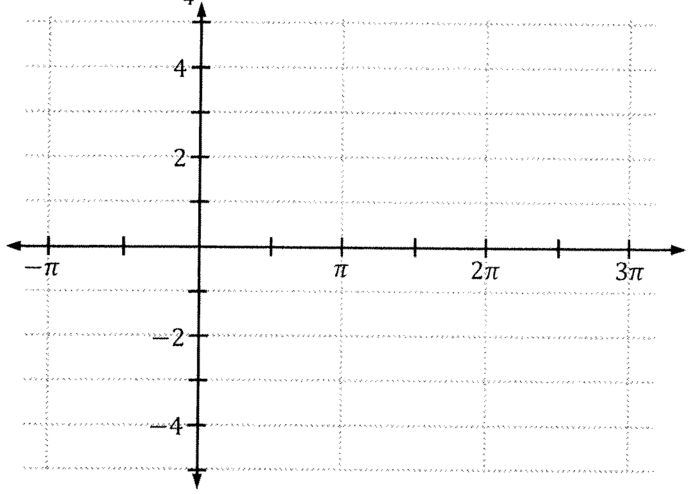
1. $y = 2\csc x$



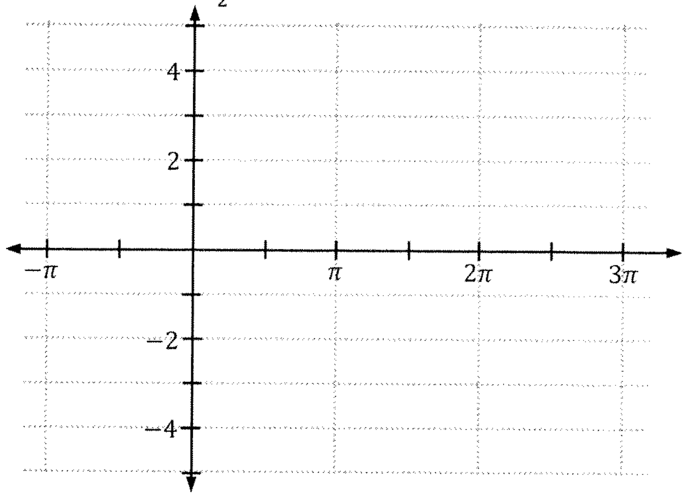
2. $y = 3\csc 2x$



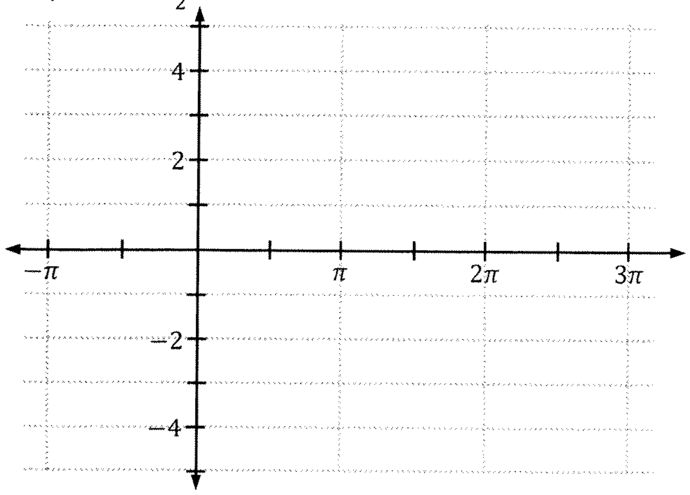
3. $y = -\csc\frac{1}{4}x$



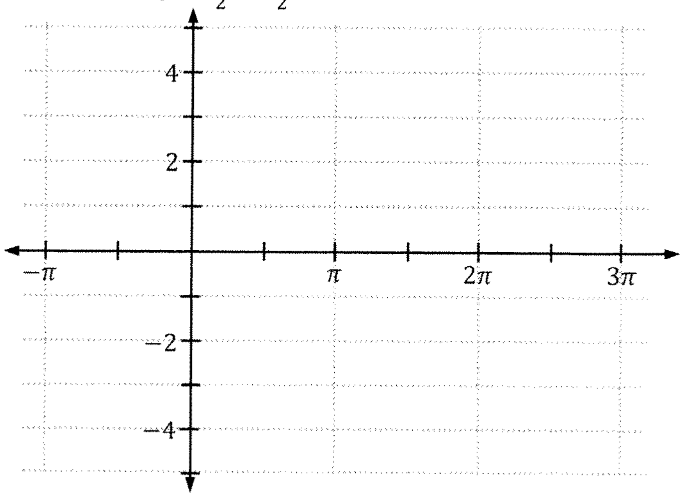
4. $y = \frac{3}{2}\csc\pi x$



5. $y = 2\csc\frac{1}{2}x$



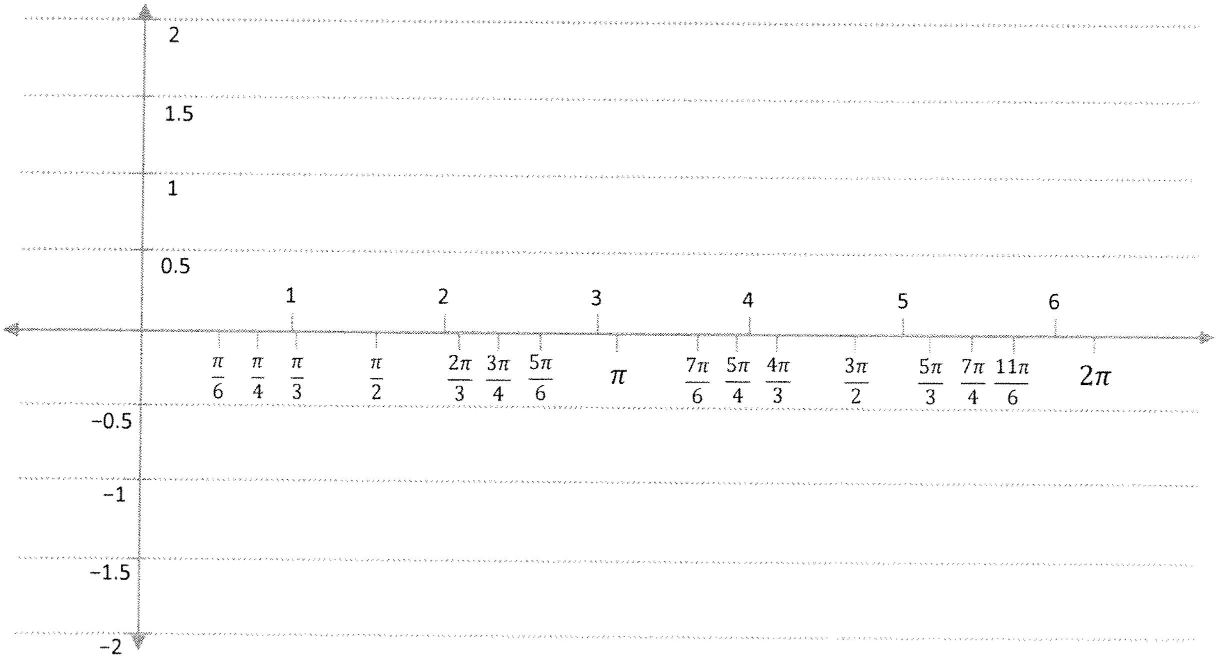
6. $y = \frac{5}{2}\csc\frac{\pi}{2}x$



II. $y = \sec x$ a) Use a calculator to fill in the table with the decimal values for the equation, $y = \cos x$. Then fill in the values for $y = \sec x$ remembering that $\csc x$ is the reciprocal of $\cos x$. After you have filled in the table of values, *lightly* sketch in $y = \cos x$ and then sketch in $y = \sec x$.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$y = \cos x$ *(decimal)*									
$y = \sec x$ *(decimal)*									

x	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$y = \cos x$ *(decimal)*									
$y = \sec x$ *(decimal)*									



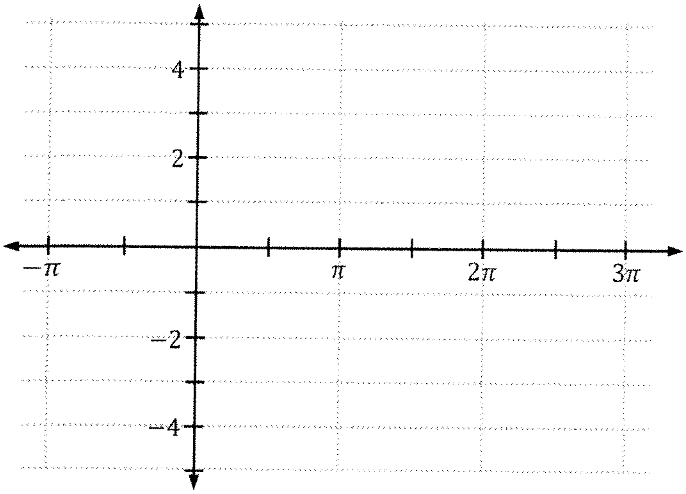
What you have just graphed is one period of the function $y = \sec x$. Answer the following questions:

b) What are some similarities between the graphs of $y = \sec x$ and $y = \csc x$?

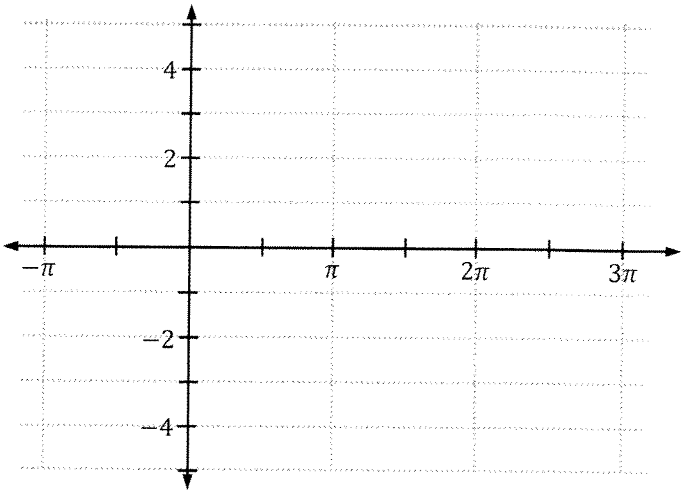
c) What are some differences between the graphs of $y = \sec x$ and $y = \csc x$?

Practice: Sketch one period of each function. Use a *lightly* drawn cosine curve to help. Label all of the critical points. Remember how the amplitude and period adjustments work with $y = \cos x$.

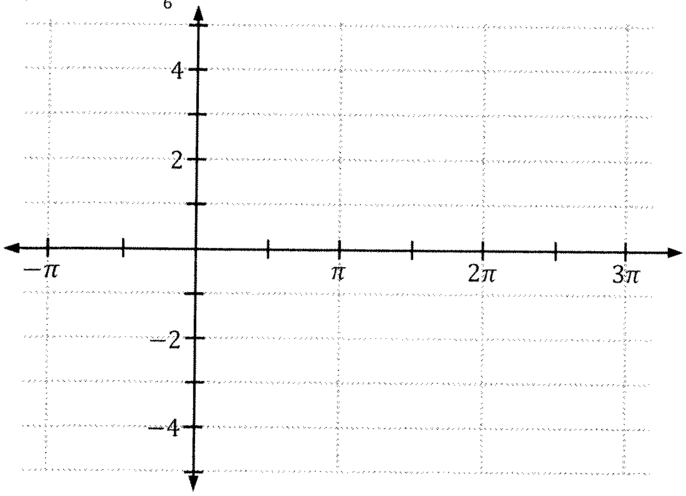
1. $y = 3\sec x$



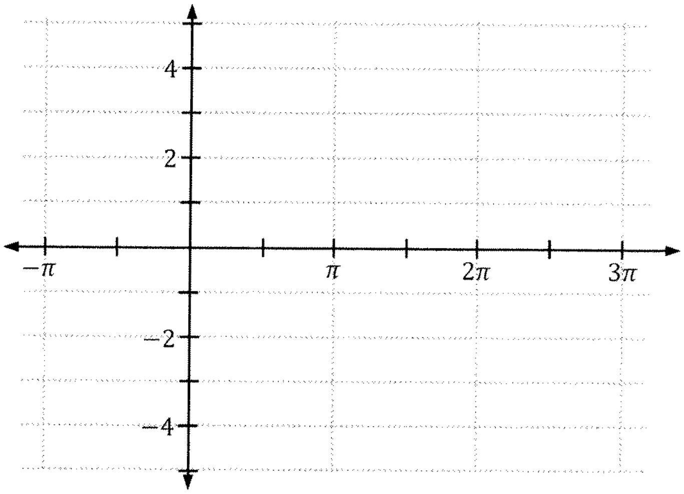
2. $y = -\frac{1}{4}\sec\frac{1}{3}x$



3. $y = 2.5\sec\frac{\pi}{6}x$

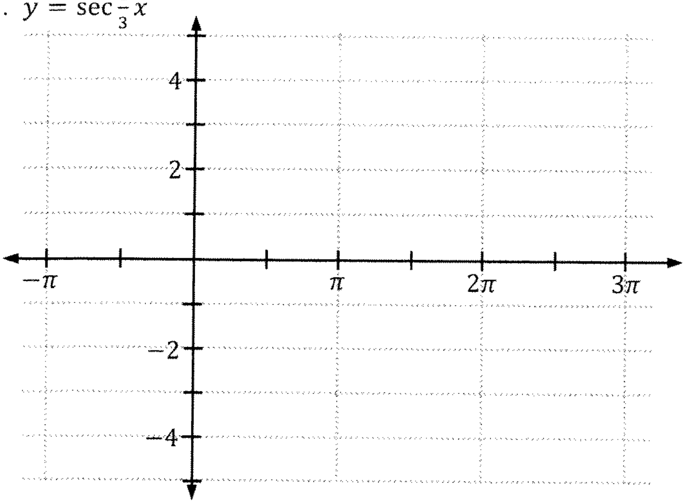


4. $y = -5\sec 4\pi x$

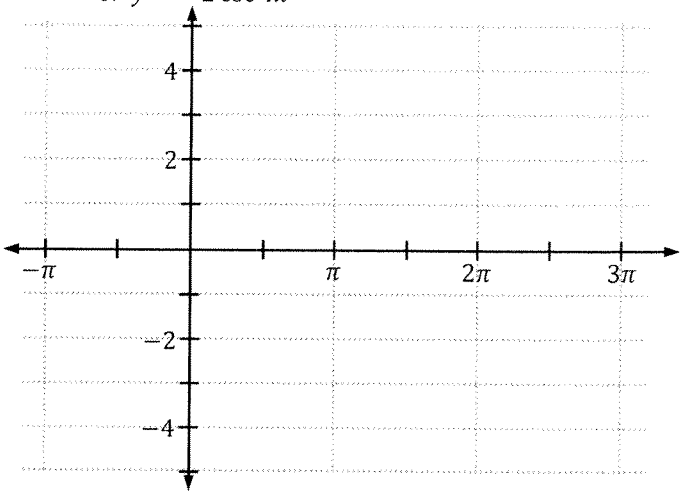


Sketch TWO periods of each function. Label all of the critical points.

5. $y = \sec\frac{1}{3}x$

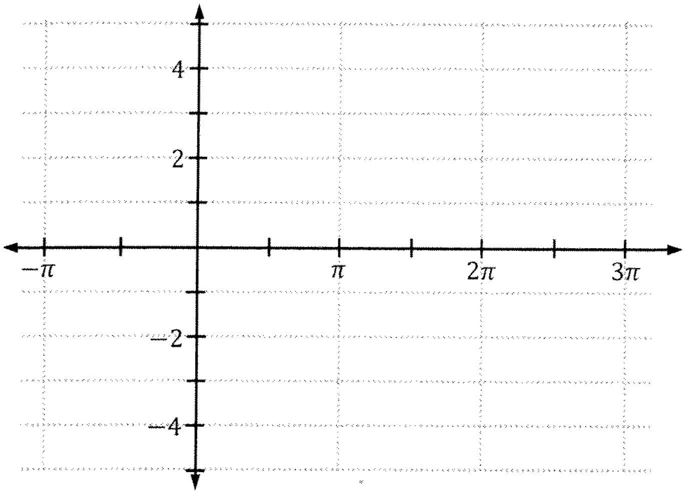


6. $y = -2\csc 4x$

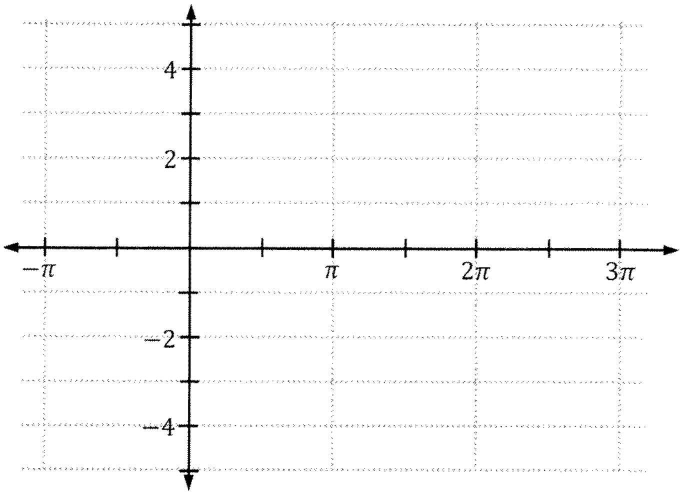


Sketch the functions on the given interval. Label all of the critical points.

7. $y = \csc 2x, -\frac{3\pi}{2} \leq x \leq \pi$



8. $y = 4\sec\frac{\pi}{2}x, -1 \leq x \leq 2$



Summer Supplement: Graphing Trig. Functions Unit
Lesson #5: Writing Trig Equations, Examples

Name:______________________________

Write an equation for a trig function with the given characteristics:

① $y = A \sin Bx$ ② $y = A \cos Bx$ ③ $y = A \sec Bx$ ④ $y = A \csc Bx$

* A = amplitude and direction * $B = \frac{2\pi}{\text{period}}$

1. A sine curve with amplitude 3 and period π.

$A = 3$; $B = \frac{2\pi}{\pi} = 2$ $\Rightarrow$ $\boxed{y = 3 \sin 2x}$

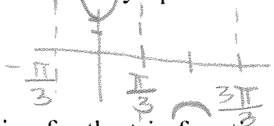
2. A cosine curve with amplitude of 1 and period $\frac{\pi}{2}$.

$A = 1$, $B = \frac{2\pi \cdot 2}{\pi/2 \cdot 2} = \frac{4\pi}{\pi} = 4$ $\Rightarrow$ $\boxed{y = 1 \cos 4x}$

3. A secant curve with amplitude 0.7 and period 5.

$A = 0.7$; $B = \frac{2\pi}{5}$ $\Rightarrow$ $\boxed{y = 0.7 \sec \frac{2\pi}{5} x}$

4. A secant curve with asymptotes at $x = \frac{\pi}{3}$ and $x = \pi$ with a y-intercept of $(0,2)$



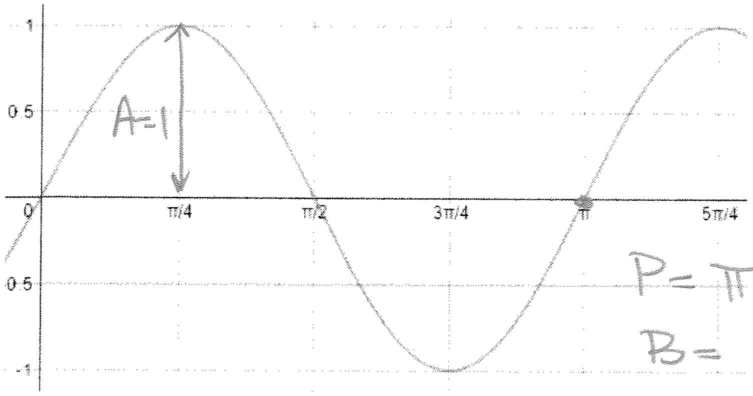
$A = 2$; $P = \frac{4\pi}{3}$

$B = \frac{2\pi \cdot 3}{\frac{4\pi}{3} \cdot 3} = \frac{6\cancel{\pi}}{4\cancel{\pi}} = \frac{3}{2}$

$\boxed{y = 2 \sec \frac{3}{2} x}$

Write an equation for the trig function with the given graph.

5.

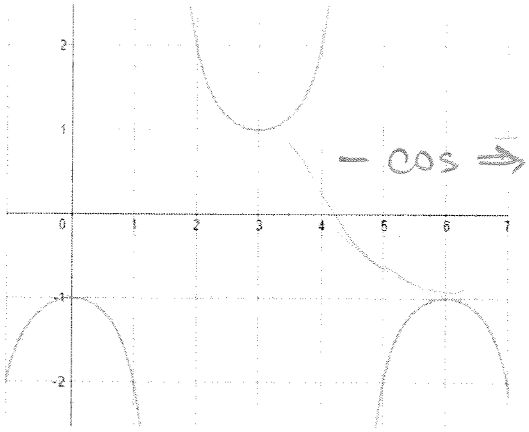


$P = \pi$

$B = \frac{2\cancel{\pi}}{\cancel{\pi}}$; $B = 2$

$\boxed{y = 1 \sin 2x}$

6.



$-\cos \Rightarrow -\text{secant}$

$A = -1$

$B = \frac{2\pi}{6} = \pi/3$

$\boxed{y = -1 \sec \frac{\pi}{3} x}$

Summer Supplement: Graphing Trig. Functions Unit
Lesson #5: Writing Trig Equations

Name:____________________________

Web Resources

Youtube: https://www.youtube.com/watch?v=clC3ld6rJVU

Write an equation for a trig function with the given characteristics:

1. A sine curve with amplitude 2 and period 4π.

2. A cosine curve with amplitude of 1 and period 6.

3. A secant curve with amplitude .5 and period π.

4. A cosecant curve with amplitude 3.5 and period $\frac{\pi}{3}$.

5. A sine curve that has a maximum at $\left(\frac{\pi}{4}, 3\right)$ and a minimum at $\left(\frac{3\pi}{4}, -3\right)$ in the first period.

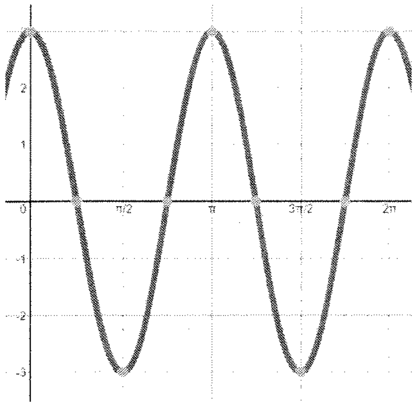
6. A cosine curve that has zeros (x-intercepts) at $(1,0)$ and $(3,0)$ in the first period with a y-intercept of $(0, k)$.

7. A sine curve with zeros at $(0,0)$ and $\left(\frac{\pi}{3}, 0\right)$ and a minimum value of $\left(\frac{\pi}{6}, -2\right)$.

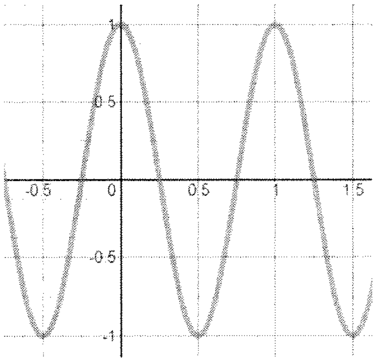
8. A secant curve with asymptotes at $x = \pi$ and $x = 3\pi$ with a y-intercept of $(0,6)$

Write an equation for the trig function with the given graph.

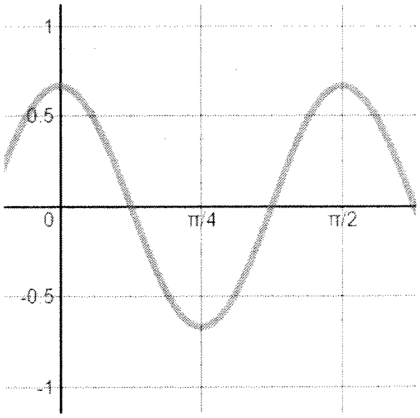
9.



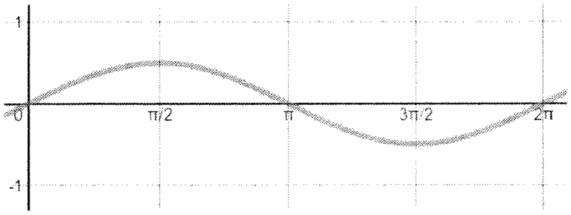
10.



11.



12.



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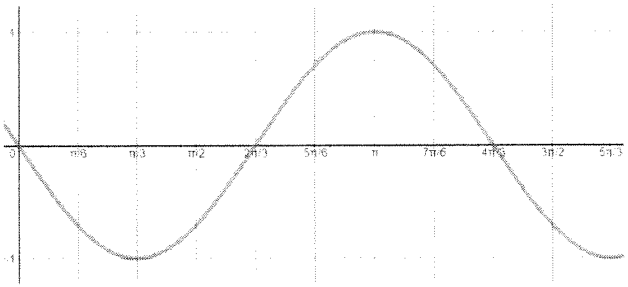
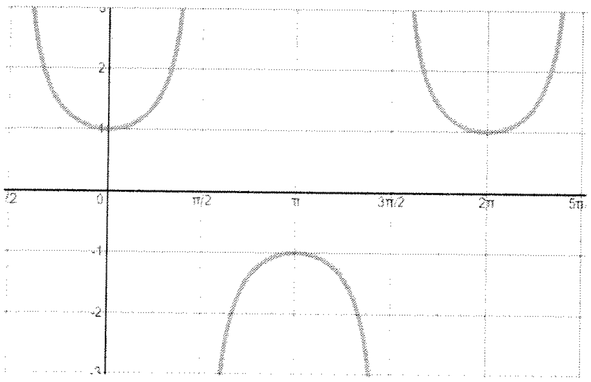
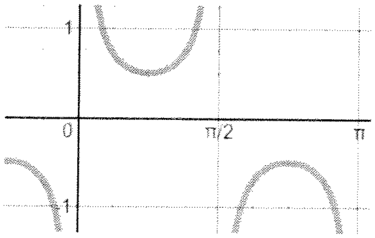


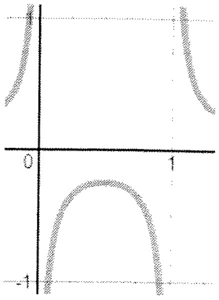
14.


3
2
1
-1
-2
-3
/2
0
π/2
π
3π/2
2π
5π

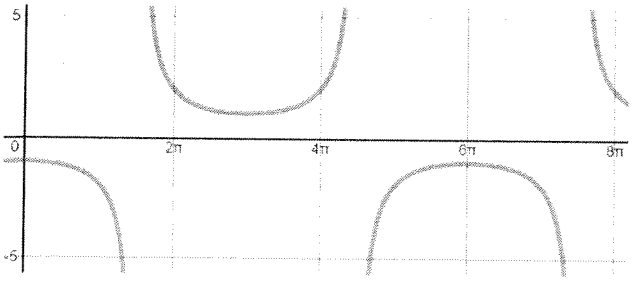
15.


1
-1
0
π/2
π

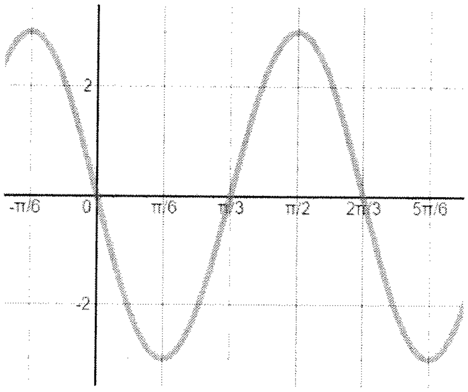
16.


1
-1
0
1

17.


5
-5
0
2π
4π
6π
8π

18.


2
-2
-π/6
0
π/6
π/3
π/2
2π/3
5π/6

Definitions / Reciprocal / Trig Identities

$\sin\theta = \frac{1}{\csc\theta}$; $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$; $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$; $\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

- $\sin^2\theta = 1 - \cos^2\theta$
- $\cos^2\theta = 1 - \sin^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

- $\tan^2\theta = \sec^2\theta - 1$
- $1 = \sec^2\theta - \tan^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

- $\cot^2\theta = \csc^2\theta - 1$
- $1 = \csc^2\theta - \cot^2\theta$

Summer Supplement: Graphing Trig. Functions Unit
Lesson #6: Simplifying using Trig Identities

Name:____________________________

Web Resources

Patrick JMT:

http://patrickjmt.com/simplifying-trigonometric-expressions-using-identities-example-1/
http://patrickjmt.com/simplifying-trigonometric-expressions-involving-fractions-ex-1/
http://patrickjmt.com/simplifying-trigonometric-expressions-involving-fractions-example-2/

Khan Academy:

https://www.khanacademy.org/math/trigonometry/trig-equations-and-identities/using-trig-identities/v/examples-using-pythagorean-identities-to-simplify-trigonometric-expressions

Simplify the expression using trigonometric identities:

1. $\sin x \sec x$

2. $\tan x \csc x$

3. $\sec x \cos x$

$\downarrow$

$\frac{1}{\cos x} \cdot \frac{\cos x}{1}$

$= \frac{\cos x}{\cos x} = \boxed{1}$

4. $\cot x \sec x$

5. $\tan x \cot x$

6. $\frac{\sin\theta}{\cos\theta}$

7. $\frac{1}{\cos\theta}$

8. $\frac{5}{\csc\theta}$

9. $\frac{\tan\theta}{\sin\theta}$

10. $\frac{\cos\theta}{\cot\theta}$

11. $\frac{\sin\theta}{\csc\theta}$

12. $\frac{\tan\theta}{\cot\theta}$

13. $1 - \sin^2\theta$

14. $\sec^2\theta - 1$

15. $\cos^2\theta - 1$

16. $1 - \csc^2\theta$

17. $4\cos^2\theta + 4\sin^2\theta$

18. $\frac{1-sin^2\theta}{tan^2\theta+1}$

19. $\frac{cos^2\theta+sin^2\theta}{csc^2\theta-1}$

20. $\sec\theta\frac{\sin\theta}{\tan\theta}$

21. $\frac{\sec^2\theta-1}{\cos^2\theta-1}$

22. $(1 - cos^2x)(\csc x)$

23. $(cos^2x)(1 - sec^2x)$

24. $\sin x - cos^2x\sin x$

25. $\cos^2 x + \tan^2 x\cos^2 x$

26. $\sin^2\theta\cos\theta + \cos^3\theta$

$$\cos\theta(\underbrace{\sin^2\theta + \cos^2\theta}_{=1})$$

$$\boxed{\cos\theta}$$

27. $\sin k\,(\csc k - \sin k)$

28. $(\cos k + \sin k)^2$

29. $(\csc \propto + 1)(\csc \propto - 1)$

30. $(\tan\beta + \sec\beta)(\tan\beta - \sec\beta)$

Summer Supplement: Graphing Trig. Functions Unit
Lesson #7: Proofs using Trig Identities

Name:____________________________________

Web Resources

Patrick JMT:

http://patrickjmt.com/proving-an-identity-example-1/
http://patrickjmt.com/proving-an-identity-example-2/

Other:

http://www.intmath.com/analytic-trigonometry/1-trigonometric-identities.php

1) $\tan\theta = \cot\theta\, tan^2\theta$

2) $(1+\tan^2 w)\cos^2 w = 1$

3) $\sin\theta = sin^3\theta + cos^2\theta \sin\theta$

4) $\frac{1-sec^2t}{sec^2t} = -sin^2t$

5) $\frac{1-\cos x}{\sin x} = \csc x\ - \cot x$

6) $\sin 2\theta \cos 2\theta\,(\tan 2\theta + \cot 2\theta) = 1$

7) $tan^2 2b + sin^2 2b + cos^2 2b = sec^2 2b$

8) $1 + sin^2k = 2 - cos^2k$

9) $sec^2x - sin^2x\, sec^2x = 1$

10) $(\sec\alpha - \cos\alpha)^2 = tan^2\alpha - sin^2\alpha$

11) $\cos A + \sin A \tan A = \sec A$

12) $\frac{\sec x \csc x}{\cot x} = \sec^2 x$

13) $(1 + \tan\theta)^2 = \sec^2\theta + 2\tan\theta$

14) $\tan B + \cot B = \sec B \csc B$

15) $\sec x - \sin x = \frac{1 - \sin x \cos x}{\cos x}$

Simplify each of the following using trig identities.

16) $\frac{\sin(-x)}{\sin\left(\frac{\pi}{2} - x\right)}$

17) $\frac{1}{\sec(-x)}$

18) $\frac{\tan\left(\frac{\pi}{2} - x\right)}{\cot(-x)}$

19) $\frac{1}{\csc\left(\frac{\pi}{2} - x\right)}$

20) $\frac{1}{\csc(-x)}$

21) $\frac{\cos(-x)}{\cot(-x)}$

22) $\frac{\cot\left(\frac{\pi}{2} - x\right)}{\sin(-x)}$

23) $\frac{\cos\left(\frac{\pi}{2} - x\right)}{\csc(-x)}$

Hints for Trig Proofs (above)

1) Factoring is sometimes helpful.
2) If the denominator of a fraction consists of only one function, break up the fraction.
3) If there are squares of functions, look for alternate forms of the Pythagorean identities.
4) Change everything to sines & cosines, then simplify.
5) Simplify by combining fractions.
6) If angles are negative, use opposite angle rules.
7) Complementary co-functions can be exchanged.
8) Avoid the introduction of radicals.